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Money weighted rate of return (MWR) versus Time weighted rate of return or (TWR)

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1. One period return

Introductory notions

Definition: A ***return*** is a gain or loss on an investment

Example: An investment of 100$ goes up to 130$.

Dollar return: 130$ –100$ = 30$ $: Units

A rate of return: $$\frac{130-100}{100}\% = 30\%$$ %: Units

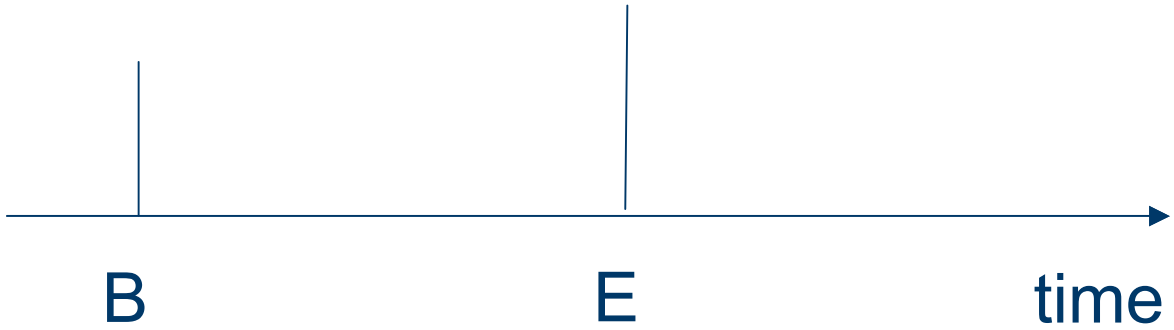
Introductory notions

B: Beginning Value, E: Ending Value

$$\frac{E-B}{B}\% = \frac{\lambda \cdot E - \lambda \cdot B}{\lambda \cdot B}\% \quad \lambda \in \mathbf{R}^1$$

- Without loss generality B = 1$
- The rate of rate does not dependent of the size of the portfolio
- There is not conclusion from the percentage rate to the $ amount

The absolute return of a portfolio



Input:

Stock	Beginning	End	Return
A	120	180	33.3%
B	100	120	20.0%
C	30	90	66.6%

The absolute return of a portfolio

Evaluation for a portfolio:

Stock	Weights	Return	Absolute Contribution
A	15%	33.3%	5%
B	25%	20.0%	5%
C	60%	66.6%	40%
Portfolio return			50%

- The return of a portfolio is equal to the weighted return of the securities
- The table shows an absolute contribution
- Distinguish between weighted and unweighted return

Decomposition of the relative return for a portfolio

$$r_p - r_B = \sum_{j=1}^{n} (w_j - m_j) \cdot r_j$$

arithmetic relative return

Example for Brinson-Hood-Beebower

		Portfolio	Benchmark	Value added	Under/ Over weight	Contrib ution
	Return					
A	-20%	15%	25%	-17.5%	-10%	2.00%
B	30%	25%	25%	32.5%	0	0.00%
C	-10%	60%	50%	-7.5%	10%	-1.00%
Portfolio Return	-1.50%					1.00%
Benchmark Return	-2.50%					
Relative Return	1.00%					

Decomposition of the relative return for a segment

$$r_p - r_B = \sum_{j=1}^{n} (w_j - m_j) \cdot r_j = \sum_{j=1}^{n} W_j \cdot R_j - \sum_{j=1}^{n} M_j \cdot B_j = \sum_{j=1}^{n} (W_j \cdot R_j - M_j \cdot B_j)$$

- On a *asset level* we have two set of weights and one set of returns
- On a *segment level* we have two set of weights and two set of returns

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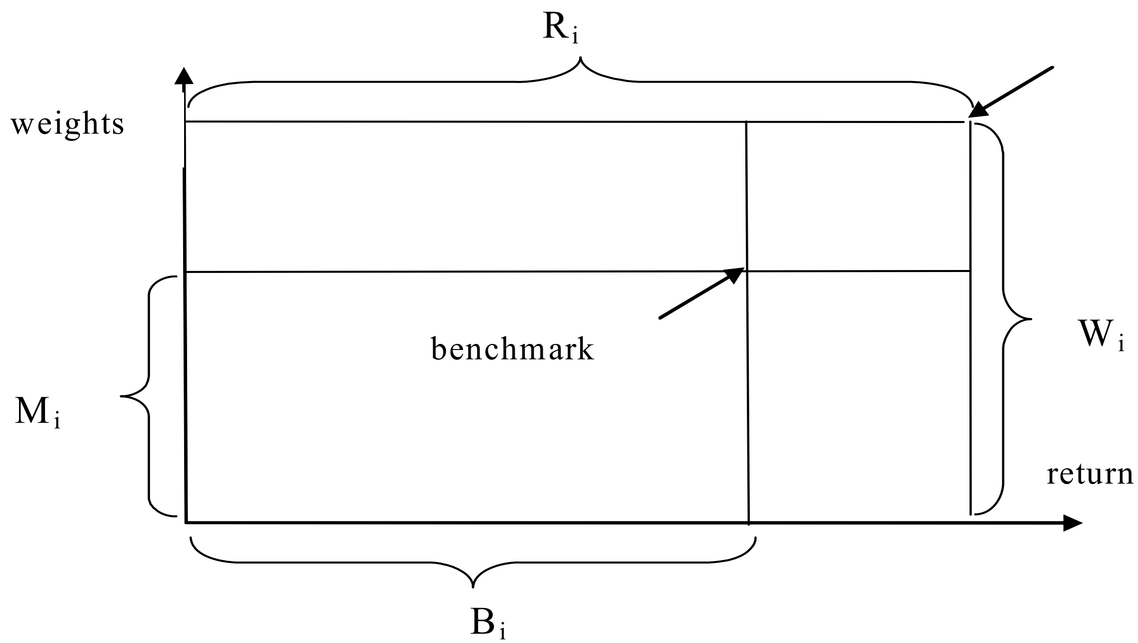
Decomposition of the relative return for a segment



$$W_j \cdot Rj_i - M_j \cdot B_j = (W_j - M_j) \cdot B_j + (R_j - B_j) \cdot M_j + (W_j - M_j) \cdot (R_j - B_j)$$

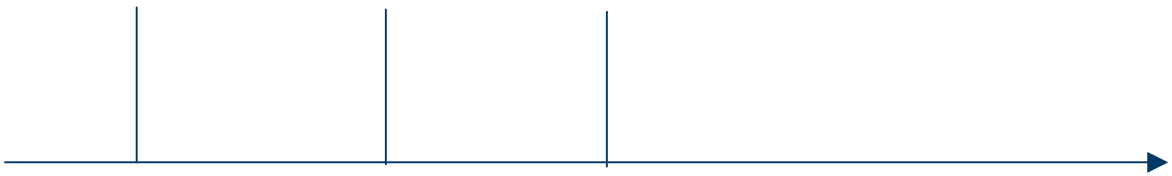
Decomposition of the relative return

$$W_j \cdot R_j - M_j \cdot B_j = \underbrace{(W_j - M_j) \cdot B_j}_{1)} + \underbrace{(R_j - B_j) \cdot M_j}_{2)} + \underbrace{(W_j - M_j) \cdot (R_j - B_j)}_{3)}$$

1) Difference in weight => Asset Allocation effect
2) Difference in return => Stock picking effect
3) Cannot be uniquely mapped => Interaction affect

2. Time weighted rate of return (TWR)

TWR on a portfolio level for 2 period



now 1 year 2 year

B_1 E_1

B_2 E_2

Cash flow C: $C = B_2 - E_1$

It is all about cash flows, the beginning and the ending value

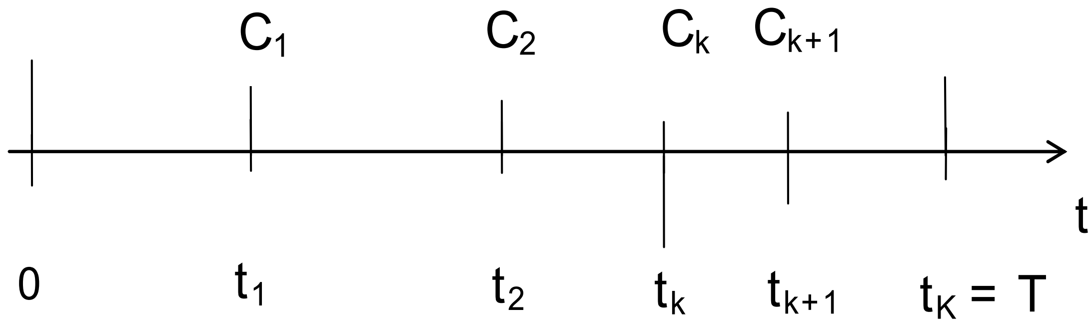
TWR on a portfolio level for multi period

$$\frac{E_1}{B_1}\frac{E_2}{B_2} - 1 = (1 + r_1)(1 + r_2) - 1$$

- The proceeds of r_1 in the first period is investment with r_2 in the second period
- TWR is an averaging method, annualizing

$$\sqrt{\frac{E_1}{B_1}\frac{E_2}{B_2}} = \sqrt{(1 + r_1)(1 + r_2)}$$

TWR on a portfolio level for multi period



$$ {}_{0}r_{P,K} = (1 + r_{P,1})(1 + r_{P,2})\ldots\ldots(1 + r_{P,K}) - 1 $$

Chain linking on a portfolio level for multi period

N_{i,t_k} Number of Units at time t_k

P_{i,t_k} Price of Security i at time t_k

$$ {}_0 r_T = \prod_{k=0}^{K-1} \frac{\sum_{i=1}^{n} N_{i,t_k} P_{i,t_{k+1}}}{\sum_{i=1}^{n} N_{i,t_k} P_{i,t_k}} - 1 $$

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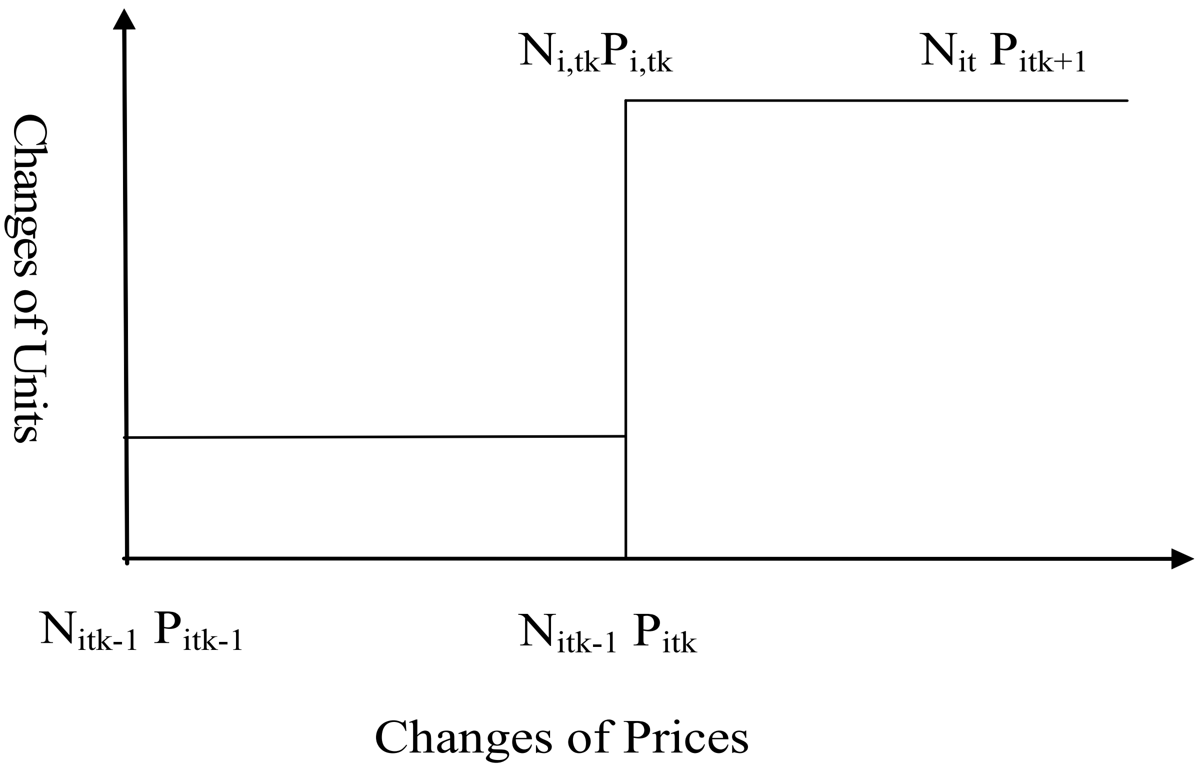
Chain linking on a portfolio level for multi period



Chain linking on a portfolio level for multi period

$$\frac{\sum_{i=1}^{n} N_{i,t_{k-1}} P_{i,t_k} \cdot \sum_{i=1}^{n} N_{i,t_k} P_{i,t_{k+1}}}{\sum_{i=1}^{n} N_{i,t_{k-1}} P_{i,t_{k-1}} \cdot \sum_{i=1}^{n} N_{i,t_k} P_{i,t_k}} \quad , k = 1,\ldots.,K - 1$$

Case 1: $\sum_{i=1}^{n} N_{i,t_{k-1}} P_{i,t_k} = \sum_{i=1}^{n} N_{i,t_k} P_{i,t_k}$

The attribution system does not to stop for calculation the return

There is an external cash flow

Chain linking on a portfolio level for multi period

Case 2:

$$\sum_{i=1}^{n} N_{i,t_{k-1}} P_{i,t_k} \neq \sum_{i=1}^{n} N_{i,t_k} P_{i,t_k}$$

There is a external cash flow

Case 2.1: $N_{i,t_k} = \lambda_k \cdot N_{i,t_{k-1}}, \lambda_k \in \mathbf{R}^1$

The attribution system does not need to stop for calculation the return period

Case 2.2: otherwise

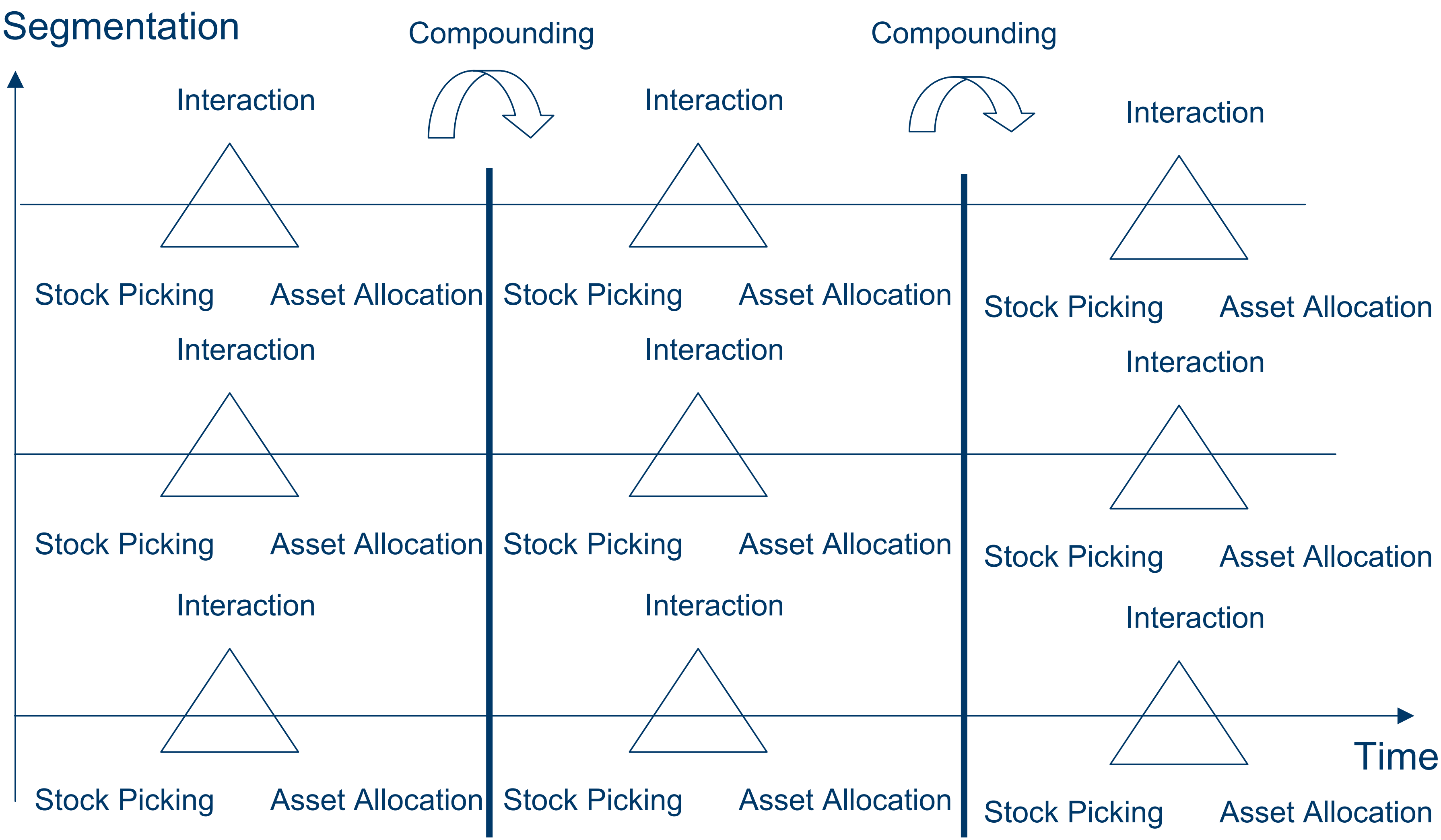
The attribution system needs to stop for calculation the return over the whole period

Properties TWR

Time-weighted rate of return (TWR) measures the return of a portfolio in a way that the return is ***insensitive*** to changes in the money invested

- TWR measures the return from a portfolio manager's perspective if he does not have control over the (external) cash flows
- TWR allows a comparison against a benchmark and across peer groups
- calculating, decomposing and reporting TWRs is common practice
- presenting TWRs is one of the key principles of the GIPS Standards

Relative Portfolio Attribution – Multi period



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Relative Portfolio Attribution – Multi period

There is a problem about decomposing the arithmetic relative return

- On segment level
- In asset allocation, stock picking and interaction effect
- A combination thereof

=> We refer to the example

3. Money weighted rate of return (MWR)

Basic Properties

$$PV_0 = \sum_{k=1}^{K-1} \frac{C_k}{(1+I)^{t_k}} + \frac{PV_T}{(1+I)^T}$$

- This equation has in general many solution
- A specific solution I is called the internal rate of rate IRR
- IRR is a MWR
- IRR is an averaging method
- MWR equal TWR is there are no cash flow
- MWR is a generalization of TWR

Properties MWR

Money-weighted rate of return (MWR) measures the return of a portfolio in a way that the return is ***sensitive*** to changes in the money invested

- MWR measures the return from a client's perspective where he does have control over the (external) cash flows
- MWR does not allow a comparison across peer groups
- MWR is best measured by the internal rate of return (IRR)
- calculating, decomposing and reporting MWRs is not common practice
- MWRs are not covered by the GIPS Standards

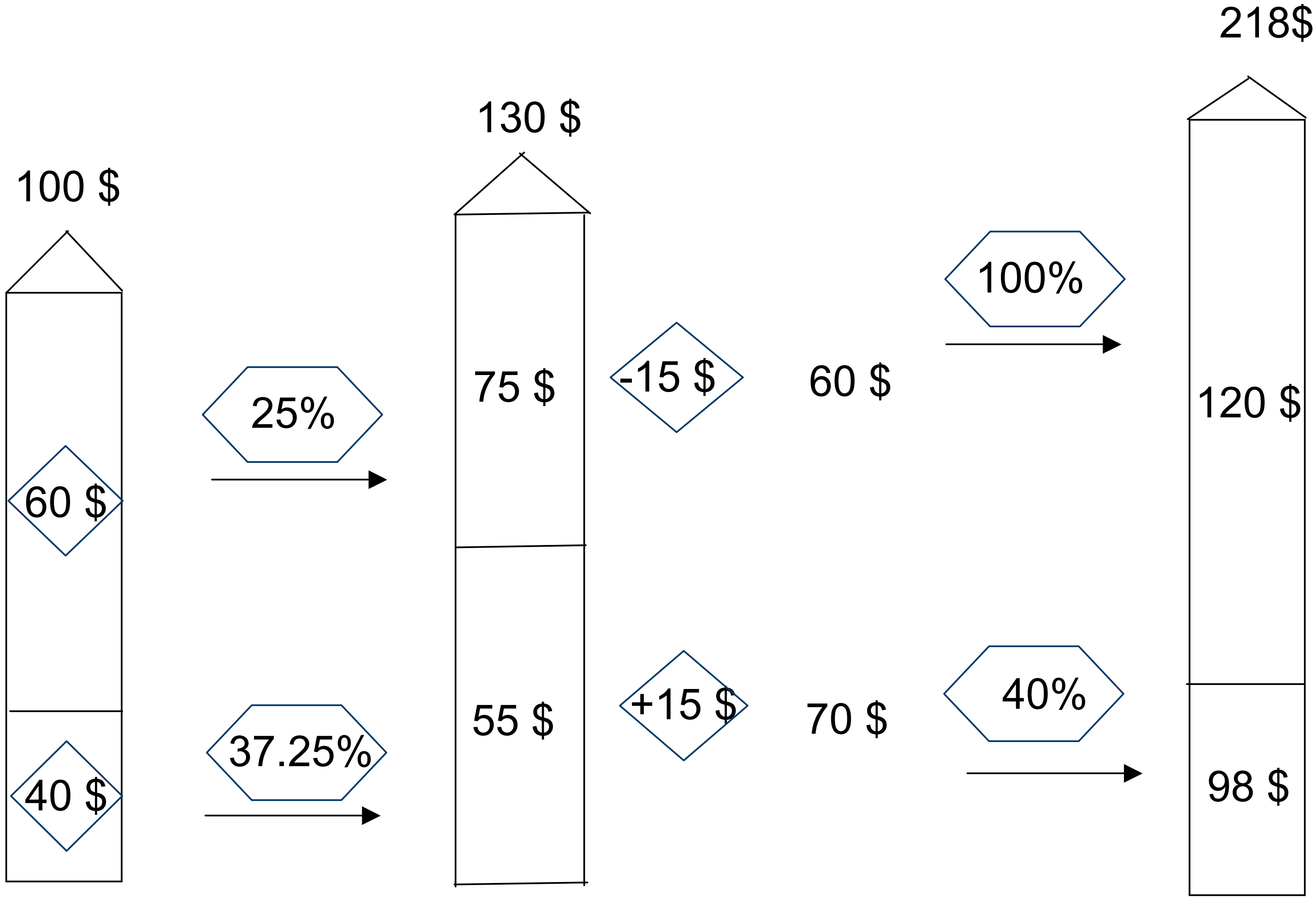
=> decomposing MWR is not addressed by the performance attribution software vendors !

4. An Example

Illustration for Performance Attribution

- We consider a Portfolio and a Benchmark (page 22)
  - with two segments
  - over two periods
- We decompose the relative return in asset allocation effect, stocking effect and interaction effect (slide 10 and 11)
- We assume an internal cash in Portfolio and Benchmark
- **There are no external cash flow in Portfolio and Benchmark => IRR = TWR**

Portfolio Value / Portfolio return



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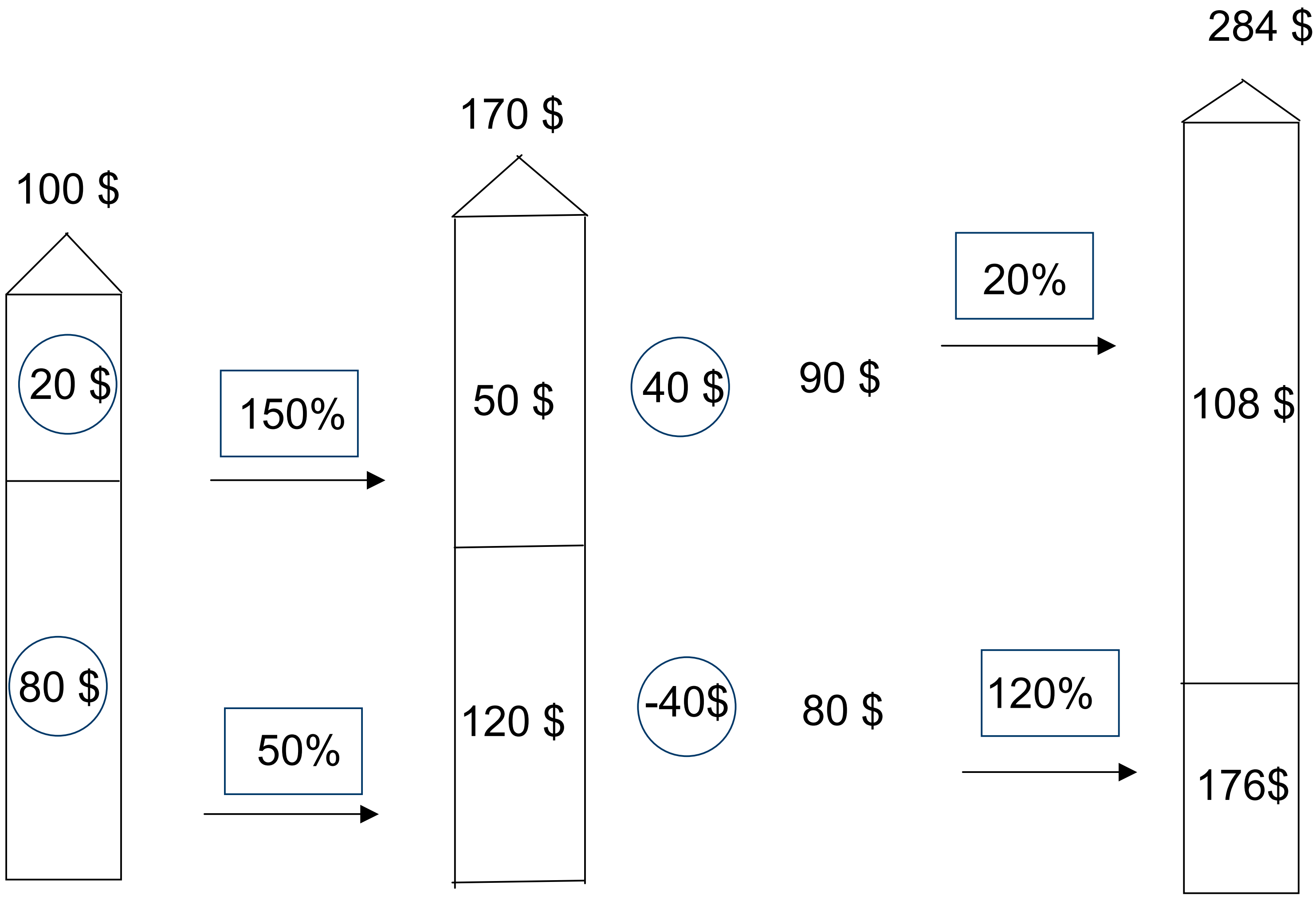
Benchmark Value / Benchmark return



3a. TWR Calculation

TWR for Portfolio (slide 28)

Periode 1	Periode 2		
$\frac{3}{5}\frac{1}{4}=\frac{3}{20}$	$\frac{6}{13}1=\frac{6}{13}$	$\left(1+\frac{3}{20}\right)\left(1+\frac{6}{13}\right)=1+\frac{3}{20}+\frac{6}{13}+\frac{3}{20}\frac{6}{13}$ $=1+0.15+0.461538+0.069231$ $=1.680769$	Segment 1
$+$	$+$		
$\frac{2}{5}\frac{3}{8}=\frac{3}{20}$	$\frac{7}{13}\frac{2}{5}=\frac{14}{65}$	$\left(1+\frac{3}{20}\right)\left(1+\frac{14}{65}\right)=1+\frac{3}{20}+\frac{14}{65}+\frac{3}{20}\frac{14}{65}$ $=1+0.15+0.215385+0.032308$ $=1.397692$	Segment 2
$=$	$=$		
$\frac{3}{10}$	$\frac{44}{65}$	$\left(1+\frac{3}{10}\right)\left(1+\frac{44}{65}\right)=1+\frac{3}{10}+\frac{44}{65}+\frac{3}{10}\frac{44}{65}$ $=1+0.30+0.676923+0.203077$ $=2.180000\ 9$	Overall

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TWR for Benchmark (slide 29)

Periode 1	Periode 2		
$\frac{1}{5}\frac{15}{10}=\frac{3}{10}$	$\frac{9}{17}\frac{1}{5}=\frac{9}{85}$	$\left(1+\frac{3}{10}\right)\left(1+\frac{9}{85}\right)=1+\frac{3}{10}+\frac{9}{85}+\frac{3}{10}\frac{9}{85}$ $=1+0.3+067058\ +0.469412$ $=1.437647$	Segment 1
$+$	$+$		
$\frac{4}{5}\frac{1}{2}=\frac{4}{10}$	$\frac{8}{17}\frac{12}{10}=\frac{48}{85}$	$\left(1+\frac{4}{10}\right)\left(1+\frac{48}{85}\right)=1+\frac{4}{10}+\frac{48}{85}+\frac{4}{10}\frac{48}{85}$ $=1+0.4+067058\ +0.469412$ $=2.190588$	Segment 2
$=$	$=$		
$\frac{7}{10}$	$\frac{57}{85}$	$\left(1+\frac{7}{10}\right)\left(1+\frac{57}{85}\right)=1+\frac{7}{10}+\frac{57}{85}+\frac{7}{10}\frac{57}{85}$ $=1+0.7+067058\ +0.469412\ =$ $=2.84$	Overall

Difference: Portfolio – Benchmark (first diffculty)

	Portfolio		Benchmark		
Segment 1	1.680769	–	1.437647	=	0.243122
Segment 2	1.397692	–	2.190588	=	- 0.7929
					≠
Overall	2.180000 9	–	2.84	=	- 0.66

- The relative return of the segment level and on portfolio do not match

Bruce Feibel on segment level

$$1+\frac{3}{20}+\frac{6}{13}+\frac{3}{20}\frac{6}{13}$$

$$\frac{3}{10}\left(\frac{6}{13}-\frac{9}{85}\right)+\frac{57}{85}\left(\frac{6}{13}-\frac{3}{10}\right)$$

$$-1-\frac{3}{10}-\frac{9}{85}-\frac{3}{10}\frac{9}{85}$$

$$1+\frac{3}{20}+\frac{14}{65}+\frac{3}{20}\frac{14}{65}$$

$$\frac{3}{10}\left(\frac{14}{65}-\frac{48}{85}\right)+\frac{57}{85}\left(\frac{3}{20}-\frac{4}{10}\right)$$

$$-1-\frac{4}{10}-\frac{48}{85}-\frac{4}{10}\frac{48}{85}$$

$$1+\frac{3}{10}+\frac{44}{65}+\frac{3}{10}\frac{44}{65}$$

$$-\frac{3}{10}\frac{57}{85}+\frac{3}{10}\frac{57}{85}$$

$$-1-\frac{7}{10}-\frac{57}{85}-\frac{7}{10}\frac{57}{85}$$

$$\frac{3}{10}\left(\frac{44}{65}-\frac{57}{85}\right)$$

$$+\frac{57}{85}\left(\frac{3}{10}-\frac{7}{10}\right)$$

Portfolio return of 1. period

Benchmark return of 2. period

Brinson-Hood-Beebower on a segment level (second difficulty)

Identity for 4 number

$$w_p \bullet r_p = w_b \bullet r_b + (w_p - w_b) \bullet r_b + (r_p - r_b) \bullet w_b + (w_p - w_b) \bullet (r_p - r_b)$$

$$\forall w_p, w_b, r_p, r_b \in \mathbf{R}^1$$

$$A = B + C - B + D - B + A - C - D + B \quad \forall A, B, C, B \in \mathbf{R}^1$$

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Brinson-Hood-Beebower

Segment 1

Periode 1

P	B
0.6	0.2
25%	150%

Periode 2

P	B
6/13	9/17
100%	20%

	Periode 1	Periode 2
Asset Allocation	(0.6 – 0.2)*150% = 0.6	(6/13 – 9/17)*20% = -0.01357
Stock Selection	(25% –150%)*0.2 = -0.25	(100% –20%)*9/17 = 0.42353
Interaction	(0.6 - 0.2)*(25% –150%) = -0.5	(6/13 – 9/17)*(25% –150%) = -0.05430

Brinson-Hood-Beebower

Segment 2

Periode 1

P	B
0.4	0.8
37.25%	50%

Periode 2

P	B
7/13	8/17
40%	120%

Asset Allocation

Periode 1: (0.4 – 0.8)*50% = -0.2

Periode 2: (7/13 – 8/17)*120% = 0.08145

Stock Selection

Periode 1: (37.5% –50%)*0.2 = -0.10

Periode 2: (40% –120%)*8/17 = -0.37647

Interaction

Periode 1: (0.4 – 0.8)*(37.5% – 50%) = 0.05

Periode 2: (7/13 – 8/17)*(40% –120%) = -0.05430

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Brinson-Hood-Beebower for cumulative Return

Effect Period 1

$$\frac{3}{10} - \frac{7}{10} = \text{-0.6 -0.25 -0.5 -0.2 -0.1 +0.05}$$

Effect Period 2

$$\frac{44}{85} - \frac{57}{85} = \text{-0.01357+42353-0.05430+0.08145-0.37647-0.05430}$$

Effect Total: — 0.66 = $\underbrace{\text{— 0.393}}_{\text{Sum}}$ $\underbrace{\text{— 0.266}}_{\text{Correction}}$

Sum Correction

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Brinson-Hood-Beebower for cumulative return

Effect Total

	A.A.	S.S.	I.A.	A.A.	S.S.	I.A.
Segment 1	0.600	-0.250	-0.500	-0.200	-0.102	0.051
Segment 2	-0.013	0.423	-0.054	0.081	-0.376	-0.054
Correction	0.984	0.132	-0.905	-0.228	-0.656	0.129

-0.500 -0.054 -3/10*0.500 -57/85*0.054

Brinson-Hood-Beebower for cumulative return

Summary:

- The correction is based on an investment assumption: portfolio return in first period times the relative return in the second period and benchmark return in second period time times the relative return in first period
- There are the same correction formulae for the relative arithmetic return as for the effects

3b. MWR Calculation

Approach of S. Illmer (Unit %)

Slide 28

Cash flow			IRR
-60	-15	120	54.4
-40	+15	98	38.9
-100	0	218	47.6

Slide 29

Cash flow			IRR
-20	40	108	52.9
-80	-40	176	75.4
-100	0	284	68.5

Approach of S. Illmer

Summary P/L

Slide 28	Segment 1	75
	Segment 2	43
	Total	118
Slide 29	Segment 1	48
	Segment 2	136
	Total	184

The average investment capital

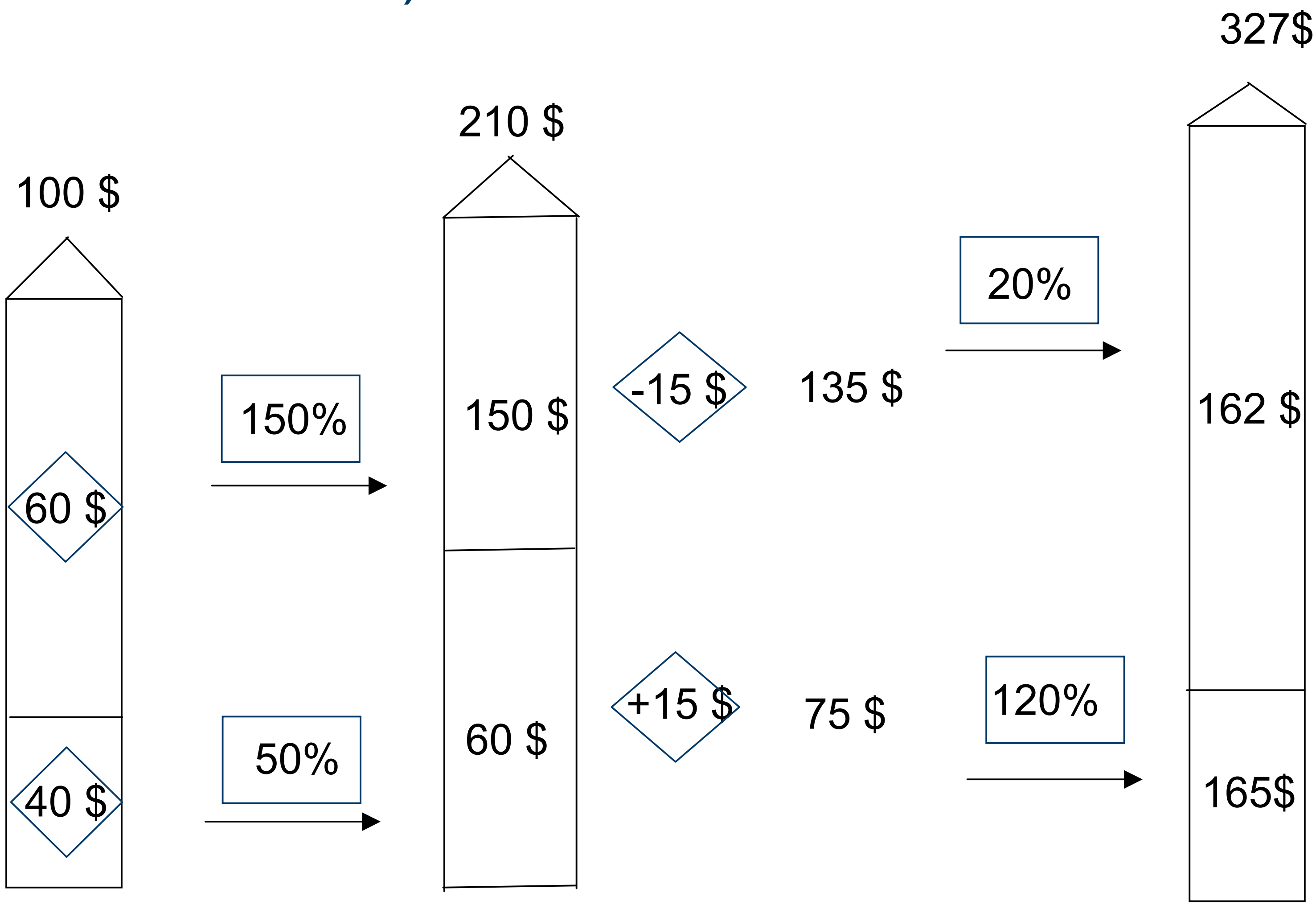
1. Step (Profit/Loss equations)

Definition of average invested capital $AIC_i = \frac{P / L_i}{I_i}$

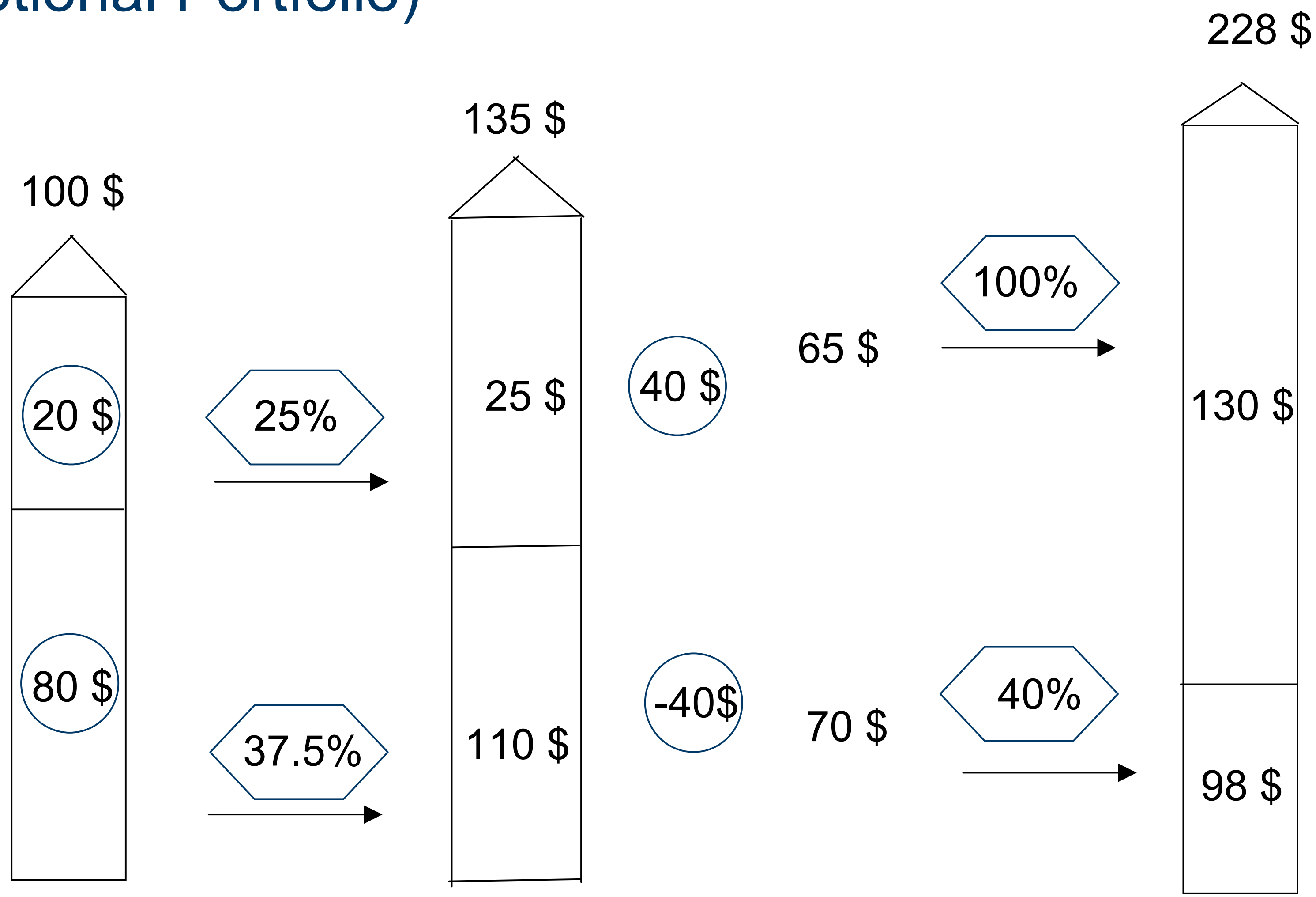
Example: 1) $AIC_i = B = \frac{E - B}{I_i}$ No cash flow

2) $AIC_i = \frac{C}{I_i}$ Perpetual annuity

Portfolio Value / Benchmark return (Asset Allocation, Notional Portfolio)



Benchmark Value/Portfolio return (Stock Picking, Notional Portfolio)



Approach of S. Illmer (Unit %)

Slide 45				Slide 46			
	Cash flow		IRR		Cash flow		IRR
-60	-15	162	77.3	-20	40	130	73.8
-40	15	165	85.2	-80	-40	98	38.4
-100	0	327	80.8	-100	0	218	50.9

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Approach of S. Illmer

Summary P/L

Slide 46	Segment 1	117
	Segment 2	110
	Total	227
Slide 47	Segment 1	70
	Segment 2	58
	Total	128

Approach of S. Illmer

$$AIC_P^1\, i_P^1 - AIC_B^1 i_B^1 = \underbrace{{}_P AIC_B^1\, {}_P i_B^1 - AIC_B^1\, i_B^1}_{\text{A.A. Segment 1}} + \underbrace{{}_B AIC_P^1\, {}_B i_P^1 - AIC_B^1 i_B^1}_{\text{S.S. Segment 1}} +$$

$$\underbrace{AIC_P^1 i_P^1 - {}_B AIC_P^1\, {}_B i_P^1 - {}_P AIC_B^1\, {}_P i_B^1 + AIC_B^1 i_B^1}_{\text{I.A. Segment 1}}$$

Approach of S. Illmer

$$AIC_P^2\, i_P^2 - AIC_B^2\, i_B^2 = {}_P AIC_B^2\, {}_P i_B^2 - AIC_B^2\, i_B^2 + {}_B AIC_P^2\, {}_B i_P^2 - AIC_B^2\, i_B^2 +$$

A.A. Segment 2   S.P. Segment 2

$$AIC_P^2\, i_P^2 - {}_B AIC_P^2\, {}_B i_P^2 - {}_P AIC_B^2\, {}_P i_B^2 + AIC_B^2\, i_B^2$$

I.A. Segment 2

$$AIC_P^T\, i_P^T - AIC_B^T\, i_B^T = {}_P AIC_B^T\, {}_P i_B^T - AIC_B^T\, i_B^T + {}_B AIC_P^T\, {}_B i_P^T - AIC_B^T\, i_B^T +$$

A.A. Total   S.S. Total

$$AIC_P^T\, i_P^T - {}_B AIC_P^T\, {}_B i_P^T - {}_P AIC_B^T\, {}_P i_B^T + AIC_B^T\, i_B^T$$

I.A. Total

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Approach of S. Illmer (Unit $)

	A.A.	S.S.	I.A.	Total
Segment 1	22	69	-64	27
Segment 2	-78	-26	11	-93
Total	-56	43	-53	-66

Approach of S. Illmer

2. Step (Return Contribution Decomposition)

$$RC_i = \frac{AIC_i}{AIC_T} I_i \quad \Rightarrow \quad I_{Total} = RC_{Total}$$

$$RC_P^1 - RC_B^1 = \underbrace{{}_P RC_B^1 - RC_B^1}_{\text{A.A. Segment 1}} + \underbrace{{}_B RC_P^1 - RC_B^1}_{\text{S.S. Segment 1}} +$$

$$\underbrace{RC_P^1 - {}_B RC_P^1 - {}_P RC_B^1 + RC_B^1 I_B^1}_{\text{I.A. Segment 1}}$$

Approach of S. Illmer

$$RC_P^2 - RC_B^2 = {}_P RC_B^2 - RC_B^2 + {}_B RC_P^2 - RC_B^2 +$$

A.A. Segment 2 S.P. Segment 2

$$RC_P^2 - {}_B RC_P^2 - {}_P RC_B^2 + RC_B^2 I_B^2$$

I.A. Segment 2

$$I_P^T - I_B^T = {}_P I_B^T - I_B^T + \quad {}_B I_P^T - I_P^T - \quad I_B^T + {}_B I_P^T - {}_P I_B^T + I_B^T$$

A.A. Total S.S. Total I.A. Total

Approach of S. Illmer (Unit decimal)

	A.A.	S.S.	I.A.	Total
Segment 1	0.1001	0.2378	-0.2139	0.1240
Segment 2	-0.2753	-0.1147	0.0573	-0.3328
Total	-0.1752	0.1230	-0.1565	-0.2087 (0.476-0.685)

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Summary

- If external cash flow of the portfolio and the benchmark are zero the IRR and the TWR are identical and as a consequence the arithmetic excess return are identical.
- The decomposition of the excess return is different even if the IRR and TWR of are the same without external cash flows.
- The complexity can be shown by a 2 segment x 2 period example.
- In a contribution the weight do not have to add to one necessarily.
- applying the yield to maturity of a portfolio